Exercise 1 Write in propositional logic: (Ecrire dans la logique propositionnelle) :

- I will only go to school if I get a cookie now. (Je vais seulement aller à l'école si je reçois un cookie maintenant.)
- John and Mary are running. (Jean et Marie courent.)
- A foreign national is entitled to social security if he has legal employment or if he has had such less than three years ago, unless he is currently also employed abroad. (Un ressortissant étranger a droit à la sécurité sociale s'il a un emploi légal ou si il a moins telle qu'il y a trois ans, à moins qu'il est aussi actuellement employés à l'étranger.)

As a method, start selecting the parts of the sentence that correspond to propositions, then identify the "logical words" in the sentence, and finally create the logical structure for the propositions.
(En tant que méthode, commencez à sélectionner les parties de la phrase qui correspondent à des propositions, puis identifiez les mots "logiques" dans la phrase, et enfin créez la structure logique des propositions).

- I will only go to school if I get a cookie now:

$$(p \to q) \wedge (q \to p)$$

  where p ="I get a cooky now" and q ="I will go to school".

- John and Mary are running:

$$p \wedge q$$

  where p ="John is running" and q ="Mary is running".

- A foreign national is entitled to social security if he has legal employment or if he has had such less than three years ago, unless he is currently also employed abroad:

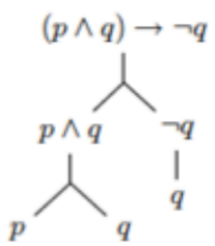
$$((p \vee q) \wedge \neg r) \to s$$

  where p ="A foreign national has legal employment", q ="A foreign national has had legal employment less then three years ago", r ="A foreign national is currently also employed abroad" and s ="A foreign national is entitled to social security".

Exercise 2 Construct a tree for the following formula: (Construire des arbres pour la formule suivante) :

- $(p \wedge q) \to \neg q$

$(p \wedge q) \to \neg q$:



Exercise 3 Construct truth tables for the following formulas: (Construire les tables de vérité pour les formules suivantes) :

- $(p \rightarrow q) \vee (q \rightarrow p)$,

$(p$	$\rightarrow$	$q)$	$\vee$	$(q$	$\rightarrow$	$p)$
0	1	0	1	0	1	0
0	1	1	1	1	0	0
1	0	0	1	0	1	1
1	1	1	1	1	1	1

- $((p \vee \neg q) \wedge r) \leftrightarrow (\neg(p \wedge r) \vee q)$.

$((p$	$\vee$	$\neg$	$q)$	$\wedge$	$r)$	$\leftrightarrow$	$(\neg$	$(p$	$\wedge$	$r)$	$\vee$	$q)$
0	1	1	0	0	0	0	1	0	0	0	1	0
0	1	1	0	1	1	1	1	0	0	1	1	0
0	0	0	1	0	0	0	1	0	0	0	1	1
0	0	0	1	0	1	0	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	0	0	1	0
1	1	1	0	1	1	0	0	1	1	1	0	0
1	1	0	1	0	0	0	1	1	0	0	1	1
1	1	0	1	1	1	1	0	1	1	1	1	1

Exercise 4 Using a truth table, determine if the two formulas (En utilisant une table de vérité, déterminer si les deux formules) :

$\neg p \rightarrow (q \vee r)$, $\neg q$

together logically imply: (ensemble, ils impliquent logiquement) :

(1) $p \wedge r$.
(2) $p \vee r$.

p	q	r	$\neg p$	$\neg q$	$q \vee r$	$\neg p \rightarrow (q \vee r)$	$p \wedge r$	$p \vee r$
0	0	0	1	1	0	0	0	0
0	0	1	1	1	1	1	0	1
0	1	0	1	0	1	1	0	0
0	1	1	1	0	1	1	0	1
1	0	0	0	1	0	1	0	1
1	0	1	0	1	1	1	1	1
1	1	0	0	0	1	1	0	1
1	1	1	0	0	1	1	1	1

Reference truth tables

Definition 2.10 (Semantics of propositional logic) A valuation V is a function from proposition letters to truth values 0 and 1. The value or meaning of complex sentences is computed from the value of basic propositions according to the following truth tables.

φ	$\neg\varphi$
0	**1**
1	**0**

φ	ψ	$\varphi \wedge \psi$	$\varphi \vee \psi$	$\varphi \rightarrow \psi$	$\varphi \leftrightarrow \psi$
0	0	**0**	**0**	**1**	**1**
0	1	**0**	**1**	**1**	**0**
1	0	**0**	**1**	**0**	**0**
1	1	**1**	**1**	**1**	**1**

(2.18)

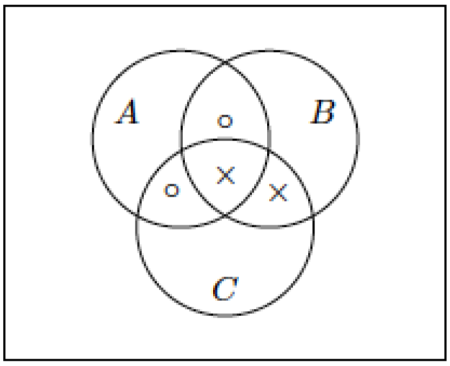
Bold-face numbers give the truth values for all relevant combinations of argument values: four in the case of connectives with two arguments, two in the case of the connective with one argument, the negation.

Verify on the Propositional Logic Calculator:
http://www.inf.unibz.it/~franconi/teaching/propcalc/

Exercise 5 Check the following syllogisms for validity, using the *situation update method* and *Venn diagrams*:
(Vérifiez la validité des syllogismes suivants, en utilisant le *situation update method* et les diagrammes de Venn) :

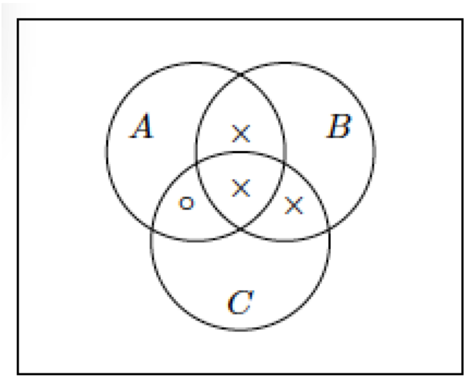
Some philosophers are Greek
No Greeks are barbarians
―――――――――
No philosophers are barbarians

→ The syllogistic pattern is not valid because it is possible to build a counter-example in which the premises are both true but the conclusion is false. This can be seen in the following diagram, where A = 'philosopher', B = 'barbarian' and C = 'greek':



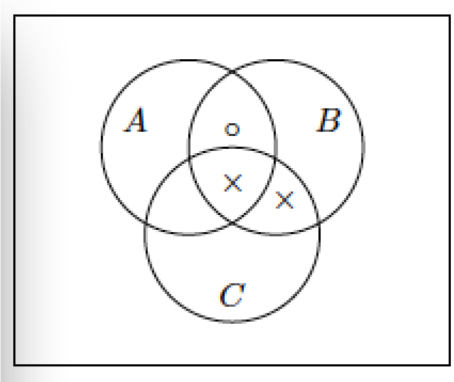
No Greeks are barbarians
No barbarians are philosophers
―――――――――
No Greeks are philosophers

→ The syllogistic pattern is not valid because it is possible to build a counter-example in which the premises are both true but the conclusion is false. This can be seen in the following diagram, where A = 'philosopher', B = 'barbarian' and C = 'greek':



No Greeks are barbarians
Some barbarians are philosophers
____
Not all philosophers are Greek

→ The syllogistic pattern is valid because after we update with the information given in the premises it is impossible for the conclusion to be false. This can be illustrated by the following diagram, where A = 'philosopher', B = 'barbarian' and C = 'greek':



Exercise 6 Give the predicate logical formulas for the following syllogistic statements: (Fournissez le formules de logique des prédicats pour les syllogismes suivants) :

(1) No B are C.

$\forall x(Bx \rightarrow \neg Cx)$ or $\neg \exists x(Bx \wedge Cx)$

(2) Some A are C.

$\exists x(Ax \wedge Cx)$

(3) Not all A are B.

$\neg \forall x(Ax \rightarrow Bx)$